MV Formulas

Carlo Angiuli

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Work, Center of Mass

$$W = \int F(x) \cdot dx$$

$$\text{General CM: } \bar{x} = \frac{\sum m_i x_i}{\sum m_i}$$

$$M_x = \rho \int_a^b \left[\frac{f(x) + g(x)}{2} \right] [f(x) - g(x)] dx$$

$$M_y = \rho \int_a^b x[f(x) - g(x)] dx$$

$$CM = \left(\frac{M_y}{m}, \frac{M_x}{m} \right)$$

$$\text{Theorem of Pappus: } V = 2\pi r A$$

(Volume of solid formed when revolving a region with area A about a line r units away from its CM.)

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\frac{d}{dx} \cosh x = \sinh x \quad \wedge \quad \frac{d}{dx} \sinh x = \cosh x$$

Cylindrical and Spherical

$$\text{Cylindrical: } x = r\cos\theta \quad \wedge \quad y = r\sin\theta \quad \wedge \quad z = z$$

$$\text{Spherical: } x = \rho\sin\phi\cos\theta \quad \wedge \quad y = \rho\sin\phi\sin\theta \quad \wedge \quad z = \rho\cos\phi$$

TNB Frame and Curvature

$$T(t) = \frac{r'(t)}{|r'(t)|}$$

$$N(t) = \frac{T'(t)}{|T'(t)|}$$

$$B(t) = T(t) \times N(t)$$

$$\kappa = \left|\frac{d\vec{T}}{ds}\right| = \frac{|T'(t)|}{|r'(t)|} = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

Partial Derivatives

$$f_x = \frac{\partial f}{\partial x}$$

$$\text{Clairaut's Theorem: } f_{xy} = f_{yx}$$

$$\text{Tangent plane: } z - z_0 = f_x(x - x_0) + f_y(y - y_0)$$

$$\text{Tangent plane: } f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

$$\nabla f = f_x\vec{\imath} + f_y\vec{\jmath} + f_z\vec{k} + ...$$

$$D_v f = \nabla f \cdot \frac{v}{|v|}$$

Second Derivatives Test

If $f_x = 0$ and $f_y = 0$, then:

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

1. $D > 0$ and $f_{xx} < 0 \rightarrow$ local maximum.
2. $D > 0$ and $f_{xx} > 0 \rightarrow$ local minimum.
3. $D > 0$ and $f_{xx} = 0 \rightarrow$ indeterminate.
4. $D < 0 \rightarrow$ saddle point.

Lagrange Multipliers

All extrema of $f(x, y, z)$ lying on the constrant $g(x, y, z) = k$ can be found by solving:

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z) \quad \wedge \quad g(x, y, z) = k$$

With two (or more) constraints:

$$\nabla f = \lambda_1 \nabla g + \lambda_2 \nabla h \quad \wedge \quad g = k_2 \quad \wedge \quad h = k_2$$

Line Integrals

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$\int_C f(x, y) ds = \int_a^b f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$\int_C F \cdot dr = \int_a^b F(r(t)) \cdot r'(t) dt$$

A vector field is conservative if it is the gradient of a scalar "potential" function.

The Fundamental Theorem of Line Integrals states that a line integral of a vector field is independent of path if and only if the integral of any closed path is 0. This occurs if and only if the vector field is conservative, in which case the integral is simply equal to the difference in the potential function between the two endpoints.

Divergence and Curl

$$\text{div } F = \nabla \cdot F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\text{curl } F = \nabla \times F = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{\text{i}} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{\text{j}} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{\text{k}}$$

$$\text{curl } \nabla f = 0$$

$$\text{div curl } F = 0$$

Green's Theorem

The line integral on a positively-oriented closed plane curve is equal to the double integral of the perpendicular curl in the bounded region.

$$\oint_C Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$A = \oint_C xdy = -\oint_C ydx$$

$$\oint_C F \cdot dr = \iint_D (\text{curl } F) \cdot k \; dA$$

$$\oint_C F \cdot n \; ds = \iint_D (\text{div } F) \; dA$$

Surface Integrals

$$A = \iint_D |r_u \times r_v| \; dA \text{ for a curve parametrized in } u \text{ and } v$$

$$A = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \; dA$$

$$\iint_S f(x,y,z) = \iint_D f(x,y,g(x,y))\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \; dA$$

$$\iint_S f(x,y,z) = \iint_D f(r(u,v))|r_u \times r_v| \; dA$$

Flux

Flux is the net amount by which a vector field flows out of a surface.

$$\Phi = \iint_S F \cdot dS = \iint_S F \cdot n \; dS$$

$$\Phi = \iint_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R \right) \; dA$$

$$\text{Divergence Theorem: } \Phi = \iint_S F \cdot dS = \iiint_E \text{div } F \; dV$$

Stokes' Theorem

The line integral on a positively-oriented closed curve on a surface is equal to the surface integral of the perpendicular portion of the curl on the bounded portion of the surface.

$$\int_C F \cdot dr = \iint_S \text{curl } F \cdot dS$$