

Determinants

Define the determinant of a collection of vectors (or a matrix) as the scalar ‘contents’ (i.e. volume) encompassed by those vectors. For example, the determinant of i, j, k is 1, the volume of the cube whose sides are i, j , and k .

Clearly, the order of the vectors can be changed without changing the magnitude of the determinant. (The sign of the determinant can be taken to represent the handedness of the axes formed by the vectors, but we will be primarily concerned with its magnitude.) If one vector is multiplied by a scalar, the determinant will be multiplied by that same scalar. If a multiple of one vector is added to another vector, the determinant remains unchanged. (What do these operations remind you of?)

The determinant of a transformation matrix represents the transformation’s volume scale factor. (Why? Hint: Think of the matrix as a set of image basis vectors.)

Some Properties of Determinants

By interpreting matrix products as compositions of transformations, the following become obvious:

$$\det(AB) = \det(A) \det(B)$$

$$\det(I) = 1$$

$$\det(A^{-1}) = \frac{1}{\det(A)}$$