Dimension

Let's play around for a minute.

$$T : \begin{cases} T(i) = 2i + 1j \\ T(j) = 1i + 3j \\ T(k) = 1i + 1j \end{cases}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2a + 1b + 1c \\ 1a + 3b + 1c \end{bmatrix}$$

Remember from earlier that $T(V_i)$ spans im T. Since $T(V_i)$ consists of three vectors in $\mathbb{R}^2$, does that mean our image has only dimension of two?

Furthermore, is there an inverse to this operation? No; it transforms three-space into a plane. Think of it as a projection: you lose information that cannot be recovered, as the inverse image of each point on the plane is a line normal to the plane passing through that point. (Recall the projection of $\mathbb{R}^3$ onto the xy-plane.)

Think of dimension as the number of degrees of freedom in a vector space.