Similarity

We already explored diagonalization, one way of shifting into a new coordinate system for a transformation. Notice that A and D are the 'same' transformation, depending on how you look at it; they do the same thing, just for different coordinate systems.

In general, any equivalent transformations like A and D are called similar transformations. We say that A is similar to D if there exists a P such that $B = P^{-1}AP$. A, B, and P must necessarily all be square matrices. (Why?)

Similarity Invariants

Similar matrices and transformations share many properties. Can you explain why the following are similarity invariants?

1. Determinant
2. Invertibility
3. Rank
4. Nullity
5. Eigenvalues
6. Eigenspace dimension for each eigenvalue
7. Characteristic polynomial

Isomorphism

Just as two transformations are similar if they look different but do the same thing, two vector spaces are isomorphic if they can be converted between freely. For this to be the case, a bijective (injective and surjective) linear transformation must exist between the two that maps between them. Any n-dimensional vector space is isomorphic to $\mathbb{R}^n$. (Why? Try polynomials, for example.)